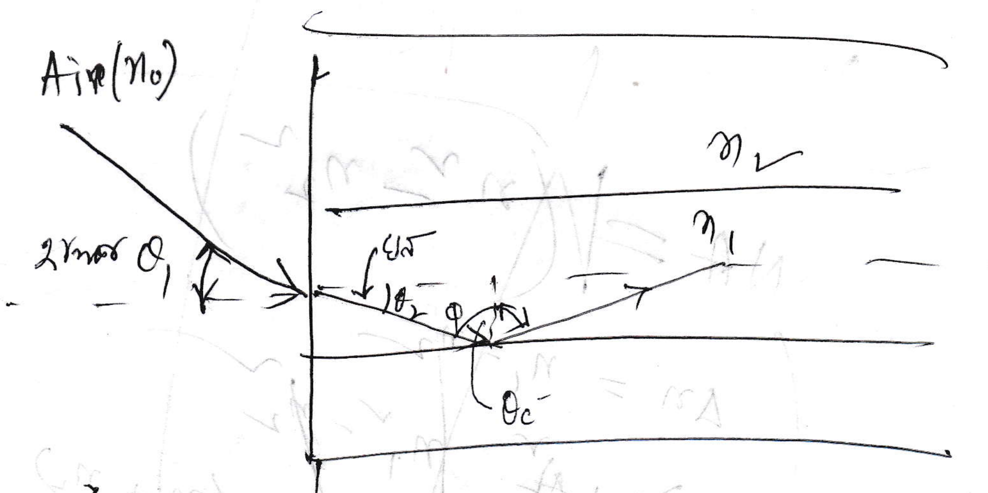
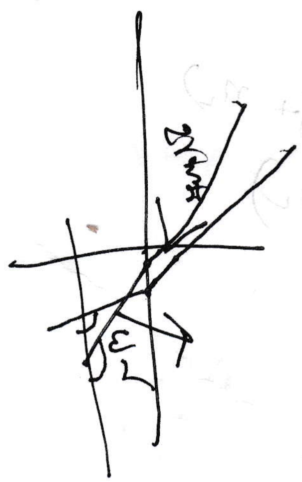
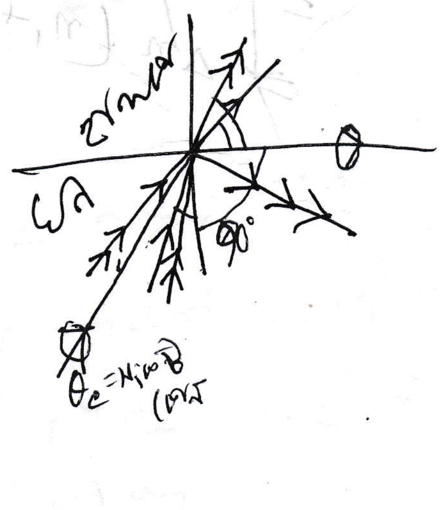
$$n_0 \sin\theta_1 = n_1 \sin\theta_2$$

$$\varphi = \frac{\pi}{2} - \theta_2$$

$$n_0 \sin\theta_1 = n_1 \sin\left(\frac{\pi}{2} - \varphi\right) = n_1 \cos\varphi = n_1 \sqrt{1 - \sin^2\varphi}$$







$\cos 0 = 1$
$\sin 90 = 1$

$n_1 \sin\theta_c = n_2 \sin 90$

$\frac{\sin\theta_c}{\sin 90} = \frac{n_2}{n_1}$

$$n_0 \sin\theta_1 = n_1\sqrt{(1-\sin^2\phi)} \quad \text{— 2.6}$$

$\sin\phi_c = \frac{n_2}{n_1}$

$\phi = \phi_c$ for limiting case of total reflection

θ_1 = acceptance angle $= \theta_a$

$$\Rightarrow n_0 \sin\theta_a = n_1\sqrt{1-\sin^2\phi_c}$$

$$= n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$= n_1\sqrt{\frac{n_1^2-n_2^2}{n_1^2}}$$

$$= \sqrt{n_1^2-n_2^2} \quad \text{— 2.7}$$

is defined as NA = numerical aperture

$$NA = n_0 \sin\theta_a = \sqrt{n_1^2-n_2^2} \quad \text{— 2.8}$$

n_0 for air $= 1$

$\therefore \sin\theta_a = \sqrt{n_1^2-n_2^2}$ over the range incidiant meridional rays will be propagated within the fiber.

$0 \le \theta_1 \le \theta_a$

$$\cancel{NA = \sqrt{n_1^2-n_2^2}}$$

$$\cancel{(NA)^2 = n_1^2-n_2^2}$$

$$\cancel{\Rightarrow 2n_1^2}$$

The NA may also be given in terms of the relative refractive index difference bet^n the core & the cladding which is defined as

~~[illegible]~~

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \quad \text{—— } \star$$

$$\simeq \frac{(n_1 - n_2)(n_1 + n_2)}{2n_1^2}$$

$$= \frac{(n_1 - n_2)\cancel{2n_1}}{\cancel{2n_1^2}} \quad \text{~~for } \Delta \leq 1\text{~~} \text{ for } \Delta \ll 1$$

এখানে n_1 উপরে
ধরা হবে

$$\cancel{\simeq \frac{n_1 - n_2}{n_1}\, n_1} \quad \text{—— } 2.9$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$\cancel{= n_1}$$

$\star$ $$2\Delta n_1^2 = n_1^2 - n_2^2$$

$$\Rightarrow \sqrt{n_1^2 - n_2^2} = \sqrt{2\Delta n_1^2}$$

$$\Rightarrow NA = n_1\sqrt{2\Delta} \quad \text{— (2)}$$

relationship is very useful measure of light-collecting ability of a fiber.

They are independent of fiber core diameter & will hold for diameters as small as 8 μm.

Example 2.1

A silica optical fiber with a core diameter large enough to be considered by ray theory analysis has a core refractive index of 1.50 and a cladding refractive index of 1.47.

Determine: (a) the critical angle at the core–cladding interface; (b) the *NA* for the fiber; (c) the acceptance angle in air for the fiber.

Solution: (a) The critical angle ϕ_c at the core–cladding interface is given by Eq. (2.2) where:

1(A)

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.47}{1.50}$$

$$= 78.5°$$

$\sin^{-1}\frac{1.57}{1.60}$ $\to 78.89°$

Done 3-9-18

(b) From Eq. (2.8) the *NA* is:

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.50^2 - 1.47^2)^{\frac{1}{2}}$$
$$= (2.25 - 2.16)^{\frac{1}{2}}$$
$$= 0.30$$

$\sqrt{(1.60^2 - 1.57^2)}$ $= .31$

(c) Considering Eq. (2.8) the acceptance angle in air θ_a is given by:

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.30$$
$$= 17.4°$$

Example 2.2

A typical relative refractive index difference for an optical fiber designed for long-distance transmission is 1%. Estimate the *NA* and the solid acceptance angle in air for the fiber when the core index is 1.46. Further, calculate the critical angle at the core–cladding interface within the fiber. It may be assumed that the concepts of geometric optics hold for the fiber.

Done 3-9-18

Solution: Using Eq. (2.10) with $\Delta = 0.01$ gives the *NA* as:

$$NA = n_1(2\Delta)^{\frac{1}{2}} = 1.46(0.02)^{\frac{1}{2}}$$
$$= 0.21$$

For small angles the solid acceptance angle in air ζ is given by:

$$\zeta \simeq \pi\theta_a^2 = \pi \sin^2 \theta_a$$

Hence from Eq. (2.8):

$$\zeta \simeq \pi (NA)^2 = \pi \times 0.04$$
$$= 0.13 \text{ rad}$$

$\because NA = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$
$\cong \sin\theta_a = NA$

Using Eq. (2.9) for the relative refractive index difference Δ gives:

$$\Delta \simeq \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

Hence

$$\frac{n_2}{n_1} = 1 - \Delta = 1 - 0.01$$

$$= 0.99$$

From Eq. (2.2) the critical angle at the core–cladding interface is:

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.99$$

$$= 81.9°$$

$\sin \theta_c = \frac{n_2}{n_1}$

meridional rays vs Skew rays

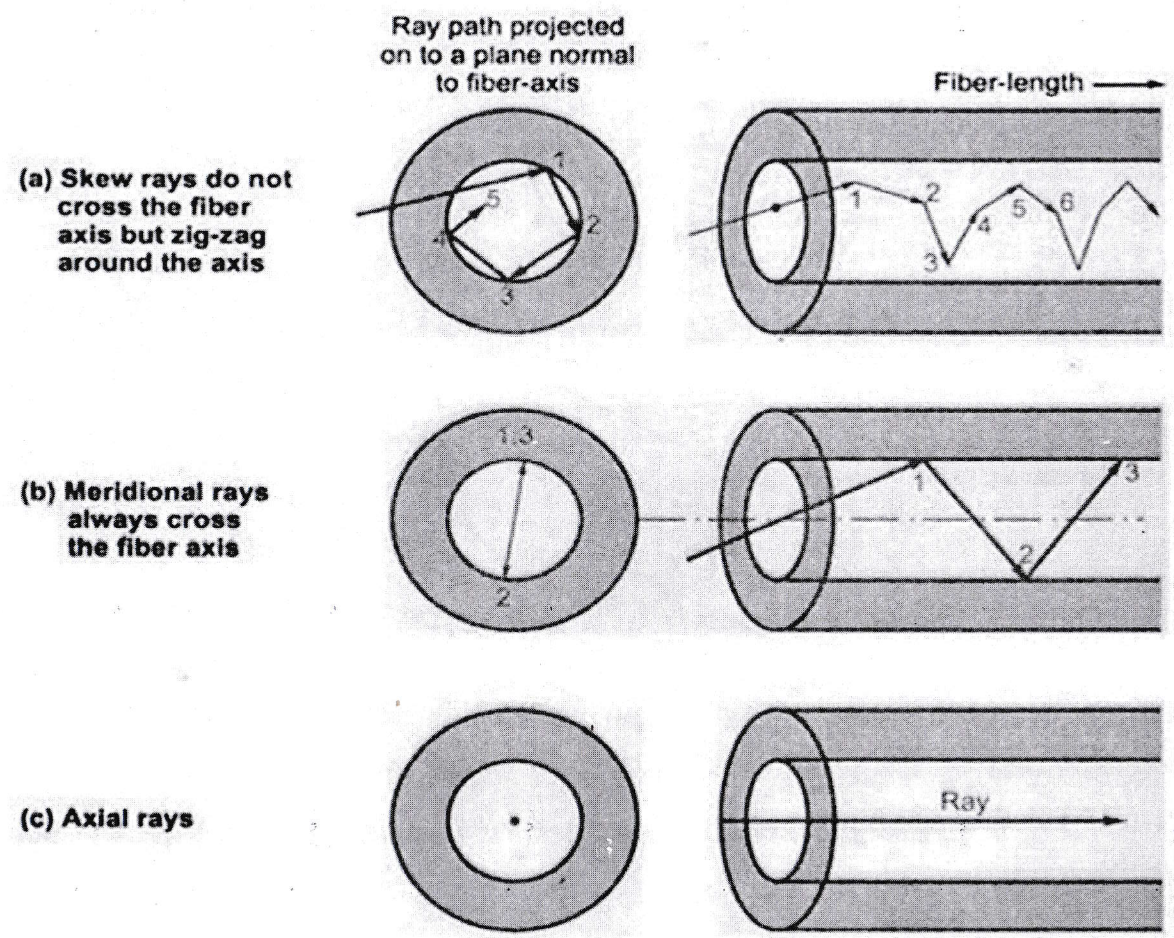


Fig. 11 Different ray propagation

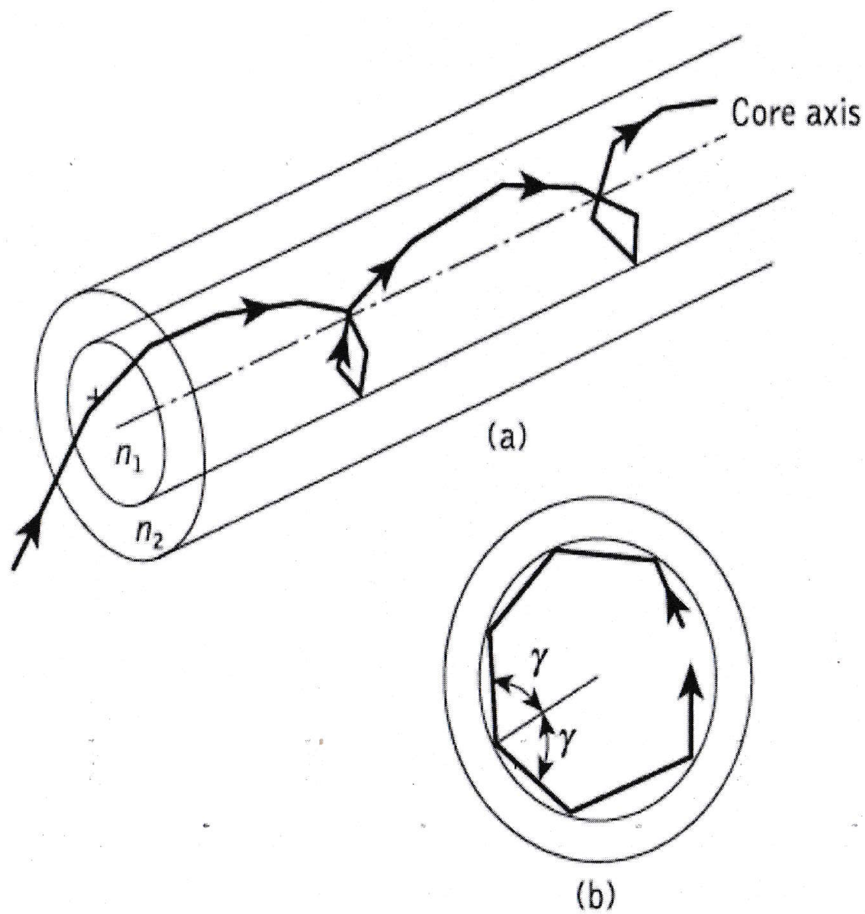


Figure 2.6 The helical path taken by a skew ray in an optical fiber: (a) skew ray path down the fiber; (b) cross-sectional view of the fiber

in Eq. (2.15) are specified for the limiting case. Thus the acceptance conditions for skew rays are:

$$n_0 \sin \theta_{as} \cos \gamma = (n_1^2 - n_2^2)^{\frac{1}{2}} = NA \tag{2.16}$$

and in the case of the fiber in air ($n_0 = 1$):

$$\sin \theta_{as} \cos \gamma = NA \tag{2.17}$$

Therefore by comparison with Eq. (2.8) derived for meridional rays, it may be noted that skew rays are accepted at larger axial angles in a given fiber than meridional rays, depending upon the value of cos γ. In fact, for meridional rays cos γ is equal to unity and θ_{as} becomes equal to θ_a. Thus although θ_a is the maximum conical half angle for the acceptance of meridional rays, it defines the minimum input angle for skew rays. Hence, as may be observed from Figure 2.6, skew rays tend to propagate only in the annular region near the outer surface of the core, and do not fully utilize the core as a transmission medium. However, they are complementary to meridional rays and increase the light-gathering capacity of the fiber. This increased light-gathering ability may be significant for large NA fibers, but for most communication design purposes the expressions given in Eqs (2.8) and (2.10) for meridional rays are considered adequate.

Example 2.3

An optical fiber in air has an NA of 0.4. Compare the acceptance angle for meridional rays with that for skew rays which change direction by 100° at each reflection.

Solution: The acceptance angle for meridional rays is given by Eq. (2.8) with $n_0 = 1$ as:

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.4$$
$$= 23.6°$$

The skew rays change direction by 100° at each reflection, therefore $\gamma = 50°$. Hence using Eq. (2.17) the acceptance angle for skew rays is:

$$\theta_{as} = \sin^{-1}\left(\frac{NA}{\cos \gamma}\right) = \sin^{-1}\left(\frac{0.4}{\cos 50°}\right)$$
$$= 38.5°$$

In this example, the acceptance angle for the skew rays is about 15° greater than the corresponding angle for meridional rays. However, it must be noted that we have only compared the acceptance angle of one particular skew ray path. When the light input to the fiber is at an angle to the fiber axis, it is possible that γ will vary from zero for meridional rays to 90° for rays which enter the fiber at the core–cladding interface giving acceptance of skew rays over a conical half angle of $\pi/2$ radians.